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Section VIII

MAGNETIC TRAPS

INVESTIGATION OF CHARGED PARTICLE MOTION IN MAGNETIC TRAPS OF ACUTE GEOMETRY

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Analysis of the characteristics of charged-particle motion in several practically encountered axisymmetric magnetic fields of acute geometry. The specific features of particle trajectories determined by numerical integration of the equations of motion are examined.

Several authors (Ref. 1-3) have investigated the motion of charged particles in acute-angle traps. However, up until recently there had been no comprehensive explanation of all the characteristics of the motion. This may be explained to a certain extent by the difficulties entailed in an analytical investigation of the magnetic fields throughout the entire trap, as well as the difficulty entailed in integrating the equations of motion. In the simplest form, such a magnetic field may be determined by the relationships /388*

$$H_r = ar; \quad H_z = -2az. \tag{1}$$

This field exists in the central region of two circular currents. The equations of particle motion in the field (1) are nonlinear, and therefore the particle trajectories may be obtained by numerical methods as a final result. However, a great deal of information on this motion may be obtained from general considerations.

This article investigates the characteristics of charged particle motion in certain specific magnetic fields whose force lines make acute angles and which have axial symmetry. Particle trajectories which are found by numerical integration of the equations of motion are discussed.

Magnetic Fields of Acute-Angle Traps

The magnetic field throughout the entire trap satisfies the equations

$$\mathrm{rot}\,\mathbf{H} = 0; \quad \mathrm{div}\,\mathbf{H} = 0$$

and may be described by means of the scalar magnetic potential Φ which satisfies the Laplace equation $\Delta\Phi = 0$. In the case of fields whose force lines /389
make acute angles and which have axial symmetry, the solution of the Laplace equation, which is symmetrical with respect to the z = 0 plane, has the following form

$$\Phi = \sum_{n=0}^{\infty} A_n I_0(k_n r) \cos k_n z, \tag{2}$$

* Numbers in the margin indicate pagination in the original foreign text.

or

$$\Phi = \sum_{n=0}^{\infty} A_n J_0(k_n r) \operatorname{ch} k_n z,$$

where the amplitude A_n and the separation of variable constants k_n may be determined by the values of the fields at the trap slits.

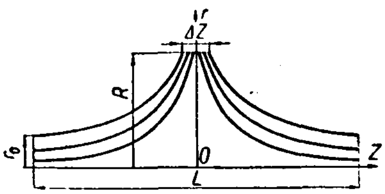

Figure 1

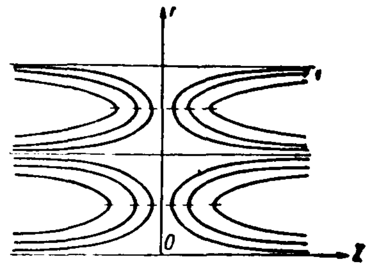

Figure 2

It is assumed in this study that throughout the entire trap the magnetic field may be described by only one component in the sums (2).

In the first case, the magnetic field then has the following form

$$\begin{aligned} H_r &= H_0 I_1(kr) \cos kz, \\ H_z &= -H_0 I_0(kr) \sin kz, \end{aligned} \tag{3}$$

and the equation of the force lines is

$$\frac{dr}{dz} = \frac{H_r}{H_z},$$

i.e.,

$$r I_1(kr) \sin kz = \text{const};$$

In the second case, we have

$$\begin{aligned} H_r &= H_0 J_1(kr) \operatorname{ch} kz, \\ H_z &= -H_0 J_0(kr) \operatorname{sh} kz \end{aligned} \tag{4}$$

with the equation of the force line

$$(kr) J_1(kr) \operatorname{sh} kz = \text{const}.$$

Figures 1 and 2 show the behavior of the force lines for magnetic traps of the first and second kind.

Traps of the first type have a magnetic field which is characteristic /390
for the customary traps of the "Picket Fence" type. The second type of trap has the form of a plane trap whose radius is not limited and whose ends are bounded by "magnetic walls" with negative magnetic field curvature. Such a type of "magnetic wall" may be produced by a system of concentric conductors with a current whose direction alternates.

The single azimuthal component (which differs from zero) of the vector potential A_ϕ has the following form for the fields (3) and (4)

$$A_\varphi = -\frac{H_0}{k} I_1(kr) \sin kz,$$

$$A_\varphi = -\frac{H_0}{k} J_1(kr) \operatorname{sh} kz.$$

In the case of kr <<1 and kz << 1, the fields (3) and (4) change into (1). However, in contrast to the field (1), they are characterized by two parameters H_0 and k, which determine the magnetic field strength in the trap mirrors as well as its geometric dimensions.

In the $kz = \pm \frac{\pi}{2}$ planes, the magnetic field has only the component H_z. If r_0 is the radius of the magnetic mirror, then the magnetic induction flux through the mirror may be written as:

$$\Phi = \frac{2\pi H_0}{k} r_0 I_1(kr_0).$$

This flux leaves through half of the circular slit which has the width Δz and which is located at a distance R from the system axis of symmetry:

$$\Phi = \frac{2\pi H_0}{k} R I_1(kR) \sin\frac{k\Delta z}{2}.$$

Consequently, the magnetic mirror radius is related to the slit width by the following relationship

$$\frac{r_0 I_1(kr_0)}{R I_1(kR)} = \sin\frac{k\Delta z}{2}. \tag{5}$$

This relationship follows from the equation of the force line.

Magnetic traps of the first type will be employed in the future, although many relationships may be readily generalized to traps with "magnetic walls".

The parameter k is related to the trap length L by the obvious equation

$$k = \frac{\pi}{L},$$

Therefore, expression (5) may be written in the following form

$$\frac{r_0 I_1\left(\frac{\pi r_0}{L}\right)}{R I_1\left(\frac{\pi R}{L}\right)} = \sin\pi\frac{\Delta z}{2L}. \tag{6}$$

Thus, the dimensions of the mirror and the circular slit may be determined in the final analysis by the relationship between the transverse and longitudinal trap dimensions. /391

We shall distinguish between two special cases: we shall call the trap elongated in the case $\frac{\pi R}{L} < 1$, and shall call it shortened in the case $\frac{\pi R}{L} > 1$.

In the case of an elongated trap, in all the formulas cited the Bessel functions may be approximated by their values in the case of small arguments. Thus, equation (6) assumes the simple form

$$\frac{r_0^2}{R^2} = \frac{\pi}{2}\frac{\Delta z}{L}. \tag{7}$$

For a greatly shortened trap, we may employ the asymptotic expression for $I_1(kR)$. In this case, assuming that $kr_0 < 1$ just as previously, we obtain

$$\frac{r_0^2}{R^2} = \frac{\pi}{2} \cdot \frac{\Delta z}{L} \frac{e^{\frac{\pi R}{L}}}{\sqrt{\frac{1}{2}\left(\frac{R}{L}\right)^3}}.$$

Consequently, for one and the same circular slit width, the elongated trap may be characterized by a smaller mirror radius as compared with the shortened trap.

The fields described above are symmetrical with respect to the z = 0 plane. Asymmetrical magnetic fields may be described by the vector potential $(0, A_\phi, 0)$ with the component A_ϕ:

$$A_\varphi = -\frac{H_0}{k}\left[I_1(kr)\sin kz + \alpha I_1\left(\frac{kr}{2}\right)\cos\frac{kz}{2}\right]$$

and

$$\begin{aligned} H_r &= H_0 I_1(kr)\cos kz - \frac{\alpha H_0}{2} I_1\left(\frac{kr}{2}\right)\sin\frac{kz}{2}, \\ H_z &= -H_0 I_0(kr)\sin kz - \frac{\alpha H_0}{2} I_0\ \frac{kr}{2}\ \cos\frac{kz}{2}. \end{aligned} \tag{8}$$

We may employ direct calculations to show that these relationships characterize a trap having the length L, which is displaced with respect to the origin by the quantity $\Delta z = \frac{\alpha\sqrt{2}}{8k}$ toward the smaller field (in this case, toward $z < 0$).

Assuming that $\alpha < 1$, we may find the magnetic field strength on the mirror axis:

On the axis of the right mirror: $\left(kz = \frac{\pi}{2} - \frac{\alpha\sqrt{2}}{8}\right)$

$$H_+ = -H_0\left(1 + \frac{\alpha}{2\sqrt{2}}\right);$$

On the axis of the left mirror: $\left(kz = -\frac{\pi}{2} - \frac{\alpha\sqrt{2}}{8k}\right)$ /392

$$H_- = H_0\left(1 - \frac{\alpha}{2\sqrt{2}}\right).$$

Thus, we have

$$\frac{H_-}{H_+} = \frac{1 - \frac{\alpha}{2\sqrt{2}}}{1 + \frac{\alpha}{2\sqrt{2}}}. \tag{9}$$

The surface of the zero z-component of the magnetic field may be found directly from the second equation of system (8). In the case of an elongated trap, this surface is a plane (within an accuracy of terms of the second order of smallness with respect to $\frac{r}{R}$) which is displaced with respect to the geometric symmetry plane of the trap by the amount

$$\Delta(kz) = \frac{\alpha}{2}\left(1 - \frac{\sqrt{2}}{4}\right). \tag{10}$$

<u>Charged Particle Motion in the Trap Magnetic Field</u>

Let us investigate particle motion in the magnetic field of an axisymmetric trap. In this case, the Lagrange function has the following form

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2\right) + \frac{e}{c} r\dot{\varphi}A_\varphi.$$

Due to the cyclicity of the coordinate ϕ, the generalized momentum P_ϕ is an integral of motion

$$P_\varphi = \frac{\partial L}{\partial\dot{\varphi}} = mr^2\dot{\varphi} + \frac{e}{c} rA_\varphi. \tag{11}$$

If the particle was produced or injected into the trap at the point (r_0, z_0), thus having the azimuthal velocity $r_0\dot{\phi}_0$, then -- according to relationship (11) -- we have

$$\dot{\varphi} = \dot{\varphi}_0\frac{r_0^2}{r^2} - \frac{e}{c}\left[rA_\varphi - (rA_\varphi)_0\right]\cdot\frac{1}{r^2}. \tag{12}$$

The Hamiltonian of the particle has the following form with allowance for equation (11):

$$H = \frac{1}{2m}\left(P_r^2 + P_z^2\right) + \frac{1}{2m}\left(\frac{P_\varphi - \frac{e}{c} rA_\varphi}{r}\right)^2. \tag{13}$$

Therefore, the formulated problem may be regarded as the problem of particle motion in the [r, z] plane in a field with the potential energy (Ref. 4)

$$U(r, z) = \frac{1}{2m}\left\{\frac{mr_0^2\dot{\varphi}_0 - \frac{e}{c}\left[rA_\varphi - (rA_\varphi)_0\right]}{r}\right\}^2. \tag{14}$$

Taking the fact into account that the Hamiltonian of the particle in the magnetic field is the integral of motion, we may find the region of the possible particle motion /393

$$\frac{1}{2m}\left\{\frac{mr_0^2\dot{\varphi}_0 - \frac{e}{c}\left[rA_\varphi - (rA_\varphi)_0\right]}{r}\right\}^2 \leqslant E_0, \tag{15}$$

where E_0 is the total particle energy (the equal sign in expression (15) determines the boundary of the region). This inequality is fulfilled everywhere in the case U = 0, i.e., in the case

$$rA_\varphi = (rA_\varphi)_0 + \frac{mc}{e} r_0^2\dot{\varphi}_0. \tag{16}$$

Thus, there is always a force line, which may be determined by equation (16), in the vicinity of which the particles may leave the trap without any hinderance.

The contour of the potential energy is determined by the coordinates of the particle injection and by the initial value $\dot{\phi}_0$.

However, it follows from formula (15) that we may assume $\dot{\phi}_0 = 0$, without restricting the generality. We must thus assume that the particle is not produced at the point $[r_0, z_0]$, but at any point of the force line (16). Therefore, the term $mr_0^2\dot{\phi}_0$ will be omitted from this point on in expression (14).

When applied to magnetic fields whose force lines make acute angles, equation (14) assumes the form

$$U(r, z) = \frac{m}{2k^2}\left(\frac{eH_0}{mc}\right)^2\left[I_1(kr)\sin kz + \frac{r_0}{r}I_1(kr_0)\right]^2, \tag{17}$$

if the particle was injected into the trap from the point $r = r_0$, $kz = -\frac{\pi}{2}$. /394

Figure 3 shows the contour of the effective potential energy (17). It may be seen from the figure that slow particles, whose energy is less than U_B, will be reflected from the opposite mirror. After several collisions with the magnetic walls, these particles will leave through the circular slit or will return to the injection location.

The particles will only leave through the opposite mirror if their energy exceeds the threshold value of U_B. Let us calculate this value. In the case $z = \frac{L}{2}$, we have

$$U(r) = \frac{m}{2k^2}\left(\frac{eH_0}{mc}\right)^2\left[I_1(kr) + \frac{r_0}{r}I_1(kr_0)\right]^2.$$

The potential energy minimum occurs in the case $r = r_1$, which may be determined by the relationship

$$k^2r_1^2I_1'(kr_1) = kr_0I_1(kr_0).$$

In the case of axial motion at the exit from the trap $kr_1 \ll 1$, and therefore we have

$$r_1 \simeq r_0.$$

Consequently, the particles will leave the trap from the opposite mirror, being injected approximately the same distance from the axis as from the trap entrance. The penetration condition may be written in the following form

$$\frac{mv_0^2}{2} > \frac{2m}{k^2}\left(\frac{eH_0}{mc}\right)^2 I_1^2(kr_0).$$

In the case when $kr_0 \ll 1$, this condition may be written as

$$R_\ell > r_0, \tag{18}$$

where $R_\ell = \frac{u_0 mc}{eH_0}$ is the Larmor particle radius in the field H_0.

Consequently, only particles whose Larmor radius is greater than the initial distance from the axis may pass through the opposite mirror. Particles injected into the trap along its axis completely penetrate the trap independently of their mass or velocity.

If particles having the same velocity but different masses are injected into the trap at a given distance from the axis, only heavy particles for which condition (18) is fulfilled will leave through the opposite mirror of the trap.

If the particle mass is such that

$$R_\ell > r_0,$$

where r_0 is the injection radius, they penetrate through the trap, without colliding with the magnetic walls, and they leave it through the opposite mirror. A bundle of particles injected with a velocity of v_0 parallel to the system axis has the following velocity of translational motion at the trap output /395

$$v = v_0 \sqrt{1 - \frac{r_0^2}{R_\ell^2}}. \qquad (19)$$

The bundle, which was rectilinear, is changed into a spiral bundle. The rotational velocity of the bundle around the system axis is

$$v_\varphi = \frac{v_0 r_0}{R_\ell}. \qquad (20)$$

The spiral step is

$$h = 2\pi R_\ell \sqrt{1 - \frac{r_0^2}{R_\ell^2}}. \qquad (21)$$

Consequently, if the magnetic field changes into a homogeneous magnetic field and a constant longitudinal field at the trap output, then the bundle of particles will move along a spiral having the radius r_0 and the step (21).

Let us now investigate particles whose Larmor radius satisfies the condition

$$R_\ell < r_0.$$

Reflected from the mirror, these particles will oscilate within the trap between the magnetic walls until they leave the trap through the circular slit or through the input mirror.

Relationship (15), which determines the oscillation boundary, assumes the following form for the case of acute angle fields

$$I_1(kr)\sin kz + \frac{r_0}{r} I_1(kr_0) = \pm kR_\ell. \qquad (22)$$

In the case of an extended trap kr << 1, the equation is simplified as follows

$$r^2 \sin kr \pm 2R_\ell r + r_0^2 = 0.$$

Thus, the equations for the boundary surfaces have the following form

$$r = \pm \frac{R_\ell}{\sin kz}\left[1 \pm \sqrt{1 - \frac{r_0^2}{R_\ell^2}\sin kz}\right]. \qquad (23)$$

Expression (23) represents a family of four surfaces, two of which correspond to the region $z < 0$, and two others correspond to the region $z > 0$. Based on the fact that the region $r < 0$ does not exist in cylindrical coordinates, it may be readily shown that for the region $z < 0$ the boundary surfaces are determined by the following relationships

$$r_1 = \frac{R_\ell}{\sin kz}\left[1 - \sqrt{1 - \frac{r_0^2}{R_\ell^2}\sin kz}\right],$$
$$r_2 = -\frac{R_\ell}{\sin kz}\left[1 + \sqrt{1 - \frac{r_0^2}{R_\ell^2}\sin kz}\right]. \qquad (24)$$

In the case $z \to 0$, the first boundary surface remains at a finite distance from the axis /396

$$r_1(0) = \frac{r_0^2}{2R_\ell}, \tag{25}$$

and the second boundary surface passes through the circular slit.

In the case $z > 0$, the equations for the corresponding surfaces have the following form

$$\begin{aligned} r_1 &= \frac{R_\ell}{\sin kz}\left[1 - \sqrt{1 - \frac{r_0^2}{R_\ell^2}\sin kz}\right], \\ r_2 &= \frac{R_\ell}{\sin kz}\left[1 + \sqrt{1 - \frac{r_0^2}{R_\ell^2}\sin kz}\right]. \end{aligned} \tag{26}$$

The relationships obtained determine the cuspidal point of the particle from the opposite lateral mirror in the case $r_0 > R_\ell$. This point is characterized by the fact that $r_1 = r_2$ -- i.e.,

$$\sin kz = \frac{R_\ell^2}{r_0^2}. \tag{27}$$

It follows from (22) that both boundary surfaces are contracted to the "eigen" particle force line when R_ℓ decreases -- i.e., to the force line where the particle was produced. However, the cuspidal point never changes in the region $z < 0$. When R_ℓ decreases, the region of possible motion degenerates into a narrow band which includes the eigen force line, in which the particle undergoes almost adiabatic motion.

Classification of Particles by Initial Parameters

Let us investigate the influence of the initial input radius of the particle r_0 and its angular velocity $\dot{\phi}_0$ on the nature of the motion for constant R_ℓ.

Particles entering the trap at a sufficiently large radius cannot intersect the plane $z = 0$, because all of their translational energy is changed into transverse energy. In the case $r = R$, the lower boundary of the region of motion intersects the circular surface of the slit in the case $z \leq 0$. According to (25), this occurs for

$$r_0 \geqslant \sqrt{2R_\ell R}.$$

Consequently, all of the particles, entering the trap with one and the same Larmor radius R_ℓ, are divided into three catagories:

(1) In the case $0 < r_0 < R_\ell$, the particles penetrate the trap, moving along a spiral around the axis, and leave through the opposite mirror;

(2) In the case $R_\ell < r_0 < \sqrt{2R_\ell R}$, the particles are reflected from the

opposite mirror and, being reflected somewhat from the trap magnetic walls, they leave it close to the surface (formed by the rotation of the eigen force line) through the circular slit, or through the input mirror; /397

(3) In the case $r_0 \geqq \sqrt{2R_\ell R}$, the particles, which do not undergo even one oscillation, move in the direction of the magnetic force line and leave the trap through the circular slit.

O. A. Lavrent'yev (Ref. 3) first divided particles into three classes. However, it should be pointed out that in the case $r_0 < R_\ell$ the particle may pass through the trap only when it arrives at the opposite mirror at the distance r_0 from the axis. If it appears at the distance $r \neq r_0$ (more or less), then -- as may be seen from the effective potential energy in the case $kz = \frac{\pi}{2}$ -- it may be reflected from the mirror. Therefore, the condition $r_0 < R_\ell$ is not absolutely definite.

The influence of the intial angular velocity of the particle upon its motion may be clarified by means of formula (12). When applied to the fields under consideration, it assumes the following form

$$\dot{\varphi} = \left(\frac{eH_0}{mc}\right)\left\{\frac{I_1(kz)\sin kz}{kr} + \frac{r_0}{kr^2}\left[\left(\frac{mc}{eH_0}\right)kr_0\dot{\varphi}_0 + I_1(kr_0)\right]\right\}. \tag{28}$$

All of the relationships obtained above are valid in the case of an initial rate of ϕ, if we substitute r_0^* determined as follows, instead of r_0:

$$kr_0^* I_1(kr_0^*) = kr_0\left[\left(\frac{mc}{eH_0}\right)kr_0\dot{\varphi}_0 + I_1(kr_0)\right]. \tag{29}$$

If $kr_0 \ll 1$ and $kr_0^* \ll 1$, then formula (29) assumes the form

$$r_0^* = r_0\sqrt{1 + 2\left(\frac{mc}{eH_0}\right)\dot{\varphi}_0}. \tag{30}$$

Consequently, for particles with $\dot{\phi}_0 > 0$ the conditions for their penetrating the trap are the same as for particles with $\dot{\phi}_0 = 0$, but with a larger input radius. In the case $\dot{\phi}_0 < 0$, they are the same as for particles with a smaller input radius. In particular, in the case $\dot{\phi}_o = -\frac{1}{2}\frac{eH_0}{mc}$, the particles penetrate the trap, without being reflected from the magnetic walls. If the magnetic field continuously changes into a homogeneous field of a solenoid at the trap output, the particles having $\dot{\phi}_0 = -\frac{1}{2}\frac{eH_0}{mc}$ will move in this field with $\dot{\phi} \approx 0$.

Motion of a Relativistic Particle in the Trap

For particles having relativistic velocities, the generalized momentum P_ϕ has the form

$$P_\varphi = \frac{mr^2\dot{\varphi}}{\sqrt{1-\beta^2}} + \frac{e}{c}rA_\varphi. \tag{31}$$

Therefore, we have /398

$$\dot{\varphi} = \frac{eH_0}{mc}(1-\beta^2)^{1/2}\left[\frac{I_1(kr)\sin kz}{kr} + \frac{kr_0 I_1(kr_0)}{k^2r^2}\right].$$

The law of conservation of energy is

$$\dot{r}^2 + \dot{z}^2 + \left(\frac{eH_0}{mc}\right)^2(1-\beta^2)\left[\frac{I_1(kz)\sin kz}{kr} + \frac{kr_0 I_1(kr_0)}{k^2r^2}\right]^2 = V_0^2. \tag{32}$$

Since all of the results given in the preceding sections were obtained from these laws of conservation, all the conclusions are valid, if we substitute the relativistic Larmor radius

$$R_\ell = \frac{\varepsilon}{cH}.$$

instead of R_ℓ. When they have sufficiently large energy, the particles may always satisfy the condition $r_0 < R_\ell$, and sufficiently relativistic particles will penetrate the trap without colliding with the magnetic walls.

A magnetic trap of this type may be utilized for transforming the linear flux of relativistic electrons into a rotating bundle with parameters determined by relationships (19) - (21). However, the small step of the spiral is only possible in the case $\frac{\varepsilon}{eH_0} \sim r_0$. Therefore, it is only possible to produce a greatly retarded rotating bundle of electrons for sufficiently strong magnetic fields. For example, for an electron energy of E = 50 Mev and r_0 = 10 cm, the the relationship $r_0 = R_\ell$ is realized for magnetic fields of $B_0 \gtrsim$ 20,000 oe.

Investigation of Particle Trajectories in a Trap

The equations for the radial and longitudinal motion have the following form

$$\ddot{r} - r\dot{\varphi}^2 = \frac{e}{mc} r\dot{\varphi}H_z,$$

$$\ddot{z} = -\frac{e}{mc} r\dot{\varphi}H_r,$$

or, with allowance for equations (3) and (28)

$$\begin{aligned} &\ddot{r} - r\left(\frac{eH_0}{mc}\right)^2\left[\frac{I_1(kr)\sin kz}{kr} + \frac{I_1(kr_0)\,kr_0}{k^2r^2}\right] \times \\ &\times \left[\left(\frac{I_1(kr)}{kr} - I_0(kr)\right)\sin kz + \frac{I_1(kr_0)\,kr_0}{k^2r^2}\right] = 0, \\ &\ddot{z} + r\left(\frac{eH_0}{mc}\right)^2\left[\frac{I_1(kz)\sin kz}{kr} + \frac{I_1(kr_0)\,kr_0}{k^2r^2}\right] I_1(kr)\cos kz = 0. \end{aligned} \tag{33}$$

In the case of an extended trap, or in the general case of axial motion, /399 when kr < 1, the equations (33) are significantly simplified

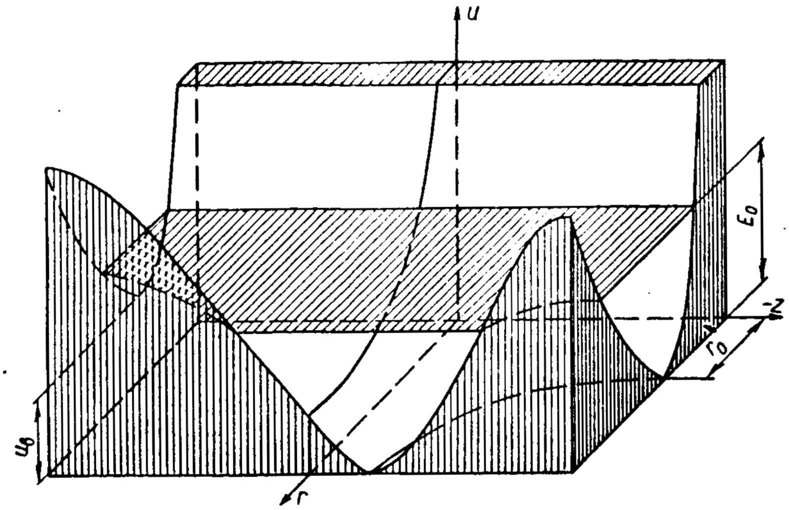

Figure 3

$$\ddot{r} + \frac{1}{4}\left(\frac{cH_0}{mc}\right)^2 r\left(\sin^2 kz - \frac{r_0^4}{r^4}\right) = 0, \tag{34}$$
$$\ddot{z} + \frac{1}{4}\left(\frac{eH_0}{mc}\right)^2 kr^2\left(\sin kz + \frac{r_0^2}{r^2}\right)\cos kz = 0.$$

Equations (34) were studied by a numerical method on a UMShN computer. The computational results are presented in the graphs shown in Figures 4-6 in dimensionless coordinates $y_1 = kr$, $y_3 = kz$, where r is the particle radial displacement from the trap axis; z -- its longitudinal coordinate. The initial parameters of the problem are

$$y_2 = kR_\perp, \text{where} R_\perp = \frac{mv_\perp c}{eH_0},$$
$$y_4 = kR_\parallel, \text{where} R_\parallel = \frac{mv_\parallel c}{eH_0}.$$

Here $v_\perp$ and $v_\parallel$ represent the particle velocity components which are perpendicular and parallel to the system axis at the moment it is injected; H_0 -- magnetic field strength on the axes of the trap mirrors. Figures 4-11 show the trajectories of particles injected into the trap through the left mirror with the parameters

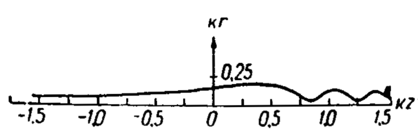

Figure 4

$$y_2 = 0; \quad y_4 = 0,1 \text{ and } kR = 0,78, \quad y_3 = -1,57.$$

In accordance with the general assumptions, one would expect that in the case $y_1 \leqq 0.1$ the particles would penetrate the trap completely, and in the case $y_1 > 0.28$ they would move toward the circular opening, without intersecting the plane $z = 0$.

Numerical computations show that direct penetration of a particle injected parallel to the axis is observed for y_1, which is somewhat less than follows from energy considerations. In the case $y_4 = 0.1$ the particles penetrate the trap completely only in the case $y_1(0) = 0.07$ (Figure 4). The particles remain in the trap for the time $\tau = 0.460$*. When $y_1(0)$ increases, the following occurs. For values of $y_1(0)$ which are close to the lower energy limit (for example, $y_1 = 0.08$), the particles move in the region $z < 0$, turning by the angle ϕ, and after intersecting the $z = 0$ plane they begin to rotate around the axis $r = 0$ (Figure 17). This rotation is asymmetrical with respect to the system axis, which leads to particle oscillations in the [r, z] plane. Being reflected from the opposite mirror, the particles return to the point of injection and leave the trap (Figures 5-8). In this case, the particles remain /400

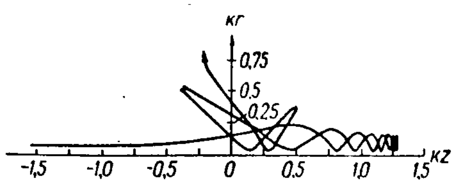

Figure 5

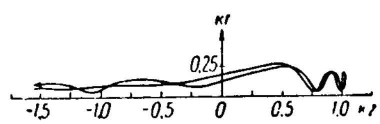

Figure 6

in the trap a relatively long period of time, due to repeated rotation in the vicinity of the mirror with a small spiral step (τ=1.236; 0.782; 0.775; 0.567..., respectively, in Figures 5-8). Repeated particle reflection is only possible when it moves toward the input mirror at a large distance ($r \gg r_0$). The outflow of the particle through the circular slit is thus more probable. The particle oscillates the greatest number of times between the magnetic walls in the case $y_1 = 0.105 - 0.110$ ($\tau = 0.757$; 1.10 respectively, in Figures 9,10). With an increase in y_1, the number of these oscillations decreases (Figure 11), and in the case $y_1 \geqq 0.28$ the particles leave (during the time $\tau = 0.407$) the circular slit, without oscillating once between the magnetic walls.

* The time τ is given in units of $10^3 \frac{mc}{eH_0}$ (Figures 4-11) and $10^6 \frac{mc}{eH_0}$ (Figures 12-16).

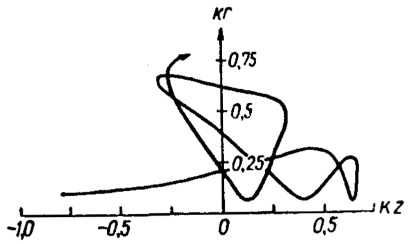

Figure 7

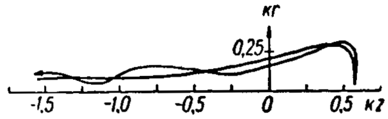

Figure 8

Thus, the particle is held in the trap only when y_1 exceeds y_4 to a very small extent. This result is more clearly apparent when particles are injected into the trap with y_4=0.0044, y_2=0, y_3= -1.57 (Figures 12-16). The value of y corresponds to a stronger magnetic field. In this case, the particle penetrates the trap only in the case y_1 = 0.0022 (Figure 12). The particle remains in the trap for the period of time τ = 0.0079. In the case $y_1 \sim y_4$ the particles are relatively "confined" (τ = 0.0243). In this case they rotate a /401 large number of times in the trap mirrors (above 100) and are reflected

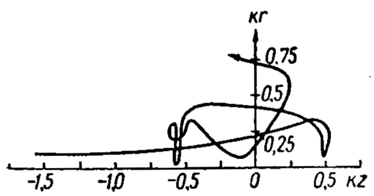

Figure 9

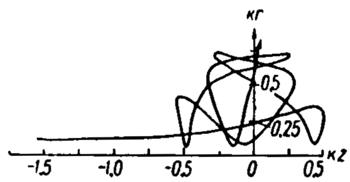

Figure 10

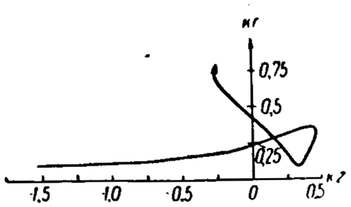

Figure 11

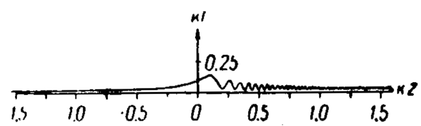

Figure 12

repeatedly from the magnetic walls (more than 23 reflections) (Figure 13). In the case y_1 = 0.01, the particle stays in the mirrors a relatively small period of time. After being reflected about 15 times from the magnetic walls, it leaves the trap through the circular slit (Figure 14, τ = 0.015). When the parameter y_1 increases further, the particle leaves the trap through the circular slit, without intersecting the plane of the zero magnetic field (Figure 16, τ = 0.0049). Figure 15 (y_1 = 0.03) shows the very small probability

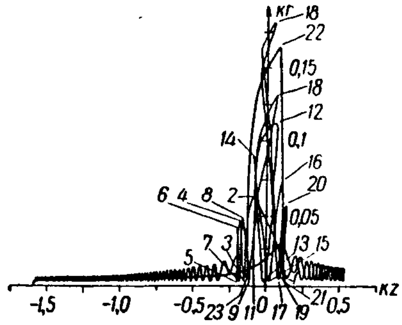

Figure 13

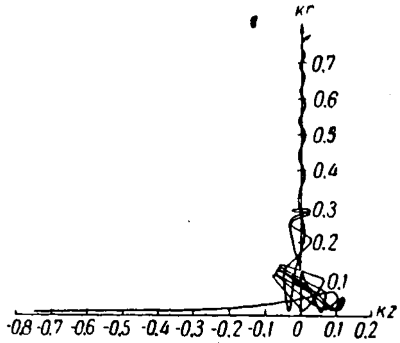

Figure 14

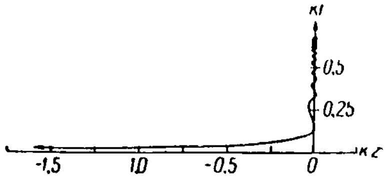

Figure 15

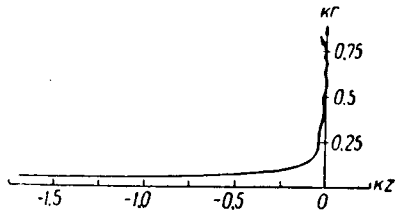

Figure 16

that a particle may be reflected from the lateral slit and leave the trap (in the time τ = 0.012), within an accuracy of its initial trajectory. Numerical integration of the particle equations of motion with other values of the magnetic fields y_4 = 0.05 and y_4 = 0.15 only substantiates the general picture of particle motion obtained from the graphs shown in Figures 4-17.

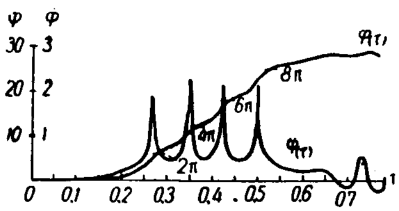

Figure 17

The following conclusions may be reached from an investigation of the trajectories. /402

1. Particles injected parallel to the system axis penetrate the trap, if the injection radius is less than 0.7 R_ℓ. In the case r_0 = 0.7 - 0.75 R_ℓ, they leave the trap, rotating around the system axis, and form a spiral with a relatively small step. The spiral axis is displaced with respect to the system geometric axis, and this displacement increases with a decrease in the distance from the axis when the injection velocity remains unchanged. The displacement increases, if the particles are not injected parallel to the system axis.

In the case 0.7 $y_4 \leq y \leq y_4$, the particles which are injected parallel to the axis are reflected from the opposite mirror, although the particles which are injected at a certain angle to the axis can penetrate the trap if they reach the opposite mirror with a radius which is close to the injection radius.

Particles reflected from the opposite mirror can remain in the trap for a period of time on the order of $\frac{100}{\omega_H}$, remaining primarily in the trap mirrors.

2. In the case $y_1 \geqq y_4$, the particles are relatively "confined". The particles can be reflected somewhat from the trap magnetic walls and leave it primarily through the circular slit. The number of reflections from the magnetic walls depends essentially on the magnetic field magnitude. In the case $y_4 = 0.1$, the particle is reflected somewhat (5-8 times) from the walls, and in the case $y_4 = 0.0044$ the number of reflections amounts to several tens (more than twenty).

3. In the case $y_1 > \sqrt{2y_4 kR}$, the particles leave the trap through the circular slit. The time that the particles remain in the trap is commensurable with the time of free flight.

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